LIGHT-MATTER COUPLING & QUANTIZED ELECTROMAGNETIC ENVIRONMENTS

CAVITY QED

Strong Coupling, Modal Decay Channels, and Dissipative Master Equations

An axiomatic guide detailing the physical bounds, coupling regimes, and open quantum dynamics of light-matter systems.

1. THE PARAMETER TRIAD & COUPLING REGIMES

Cavity Quantum Electrodynamics (Cavity QED) studies the quantum mechanical interactions between localized emitters (atoms or superconducting qubits) and the discrete electromagnetic modes of a resonant cavity. The system's behavior is dictated entirely by three physical frequency parameters:

Vacuum Coupling Rate (g):	Speed of energy swap between emitter and cavity mode.	$g = \vec{d} \cdot \vec{\mathcal{E}}_0/\hbar$
Cavity Decay Rate (κ):	Photon loss rate leaking out of the mirror boundary.	$\kappa = \omega_r/Q$
Emitter Dephasing Rate (γ):	Non-radiative relaxation and phase decoherence rate.	$\gamma = 1/T_2$

The Strong Coupling Threshold: The system enters the **Strong Coupling Regime** when the coherent transfer speed outpaces all dissipation channels:

$$g \gg (\kappa, \gamma) \qquad \Longrightarrow \qquad \mathcal{C} = \frac{g^2}{\kappa\gamma} \gg 1 \quad (\textbf{Cooperativity Criterion})$$

In this domain, energy is reversibly swapped back and forth between light and matter before any decoherence event occurs.

2. DISSIPATIVE OPEN SYSTEMS: THE LINDBLAD MASTER EQUATION

Because a physical cavity is never perfectly isolated, it must be modeled as an open quantum system. The full density matrix ρ of the coupled system evolves under the **Gorini-Kossakowski-Sudarshan-Lindblad (GKSL)** master equation:

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_{\text{JC}}, \rho] + \kappa\mathcal{D}[a]\rho + \gamma\mathcal{D}[\sigma_-]\rho$$

Where H_{JC} is the coherent Jaynes-Cummings Hamiltonian, and $\mathcal{D}[O]\rho = O\rho O^\dagger - \frac{1}{2}\{O^\dagger O, \rho\}$ is the Lindblad dissipative superoperator mapping explicit environmental relaxation channels into the matrix.

3. THE WEAK COUPLING LIMIT: THE PURCELL REGIME

When the cavity mirrors are lossy ($\kappa \gg g$), coherent oscillations are suppressed. Instead, the cavity alters the local density of optical states (LDOS), permanently locking system dynamics to the irreversible decay phase.

THE PURCELL SPONTANEOUS EMISSION BOOST

Evaluating the coupled state equations using Fermi's Golden Rule maps an accelerated, one-way spontaneous emission rate Γ_{cav} into the cavity channel:

$$\Gamma_{\text{cav}} = \frac{4g^2}{\kappa} = F_P\Gamma_{\text{bulk}} \qquad \text{where} \qquad F_P = \frac{3}{4\pi^2}\left(\frac{\lambda}{n}\right)^3\left(\frac{Q}{V_{\text{mode}}}\right)$$

The Engineering Edge: Maximizing the Purcell Factor (F_P) requires minimizing the physical mode volume V_{mode}, packing vacuum fluctuations into ultra-confined nanoscale footprints to force rapid state transitions.

4. SCALING TO CIRCUIT QED (CQED)

While atomic Cavity QED isolates natural gas-phase atoms matching optical cavities, **Circuit QED** replaces them with macroscopic superconducting circuits coupled to on-chip microwave transmission line resonators.

- **Dipole Moment Scaling:** Synthetic solid-state artificial atoms possess electric dipole moments $\vec{d}$ that are 10^3 to 10^4 times larger than natural alkali atoms.
- **Consequence:** The field achieves ultra-strong coupling regimes ($g/\omega_r > 0.1$) effortlessly on chip architectures, serving as the default hardware bus for multi-qubit entangling routines.