CLASSICAL MECHANICS & ELECTRO-DYNAMICS FOUNDATIONS

CLASSICAL PRECURSORS TO QUANTUM

Waves, Resonance, Hamiltonians, and Boundary Anomalies

An axiomatic guide mapping macroscopic physical phenomena to the mathematical architectures of quantum mechanics.

1. WAVE SUPERPOSITION & INTERFERENCE (THE KINEMATIC FOUNDATION)

Quantum mechanics structures state spaces using complex probability waves. Understanding quantum computing speedups requires analyzing classical wave kinematics under the **Superposition Principle**, where overlapping wave fields sum linearly.

The Classical Wave Equation & Phase Dynamics: The profile of two interfering harmonic waves sharing an amplitude E_0 and wavenumber k is modeled as:

$$E_{\text{total}}(x,t) = E_0\cos(kx-\omega t) + E_0\cos(kx-\omega t+\phi) = 2E_0\cos\left(\frac{\phi}{2}\right)\cos\left(kx-\omega t+\frac{\phi}{2}\right)$$

Resulting Intensity ($I \propto |E|^2$):

$$I_{\text{total}} = 4I_0\cos^2\left(\frac{\phi}{2}\right)$$

- **Constructive Interference:** $\phi = 2n\pi \implies I_{\max} = 4I_0$. Amplitudes reinforce each other.
- **Destructive Interference:** $\phi = (2n+1)\pi \implies I_{\min} = 0$. Waves cancel out completely.
- **Quantum Link:** This mathematical baseline maps directly to **Grover's Algorithm**, where logic gates manipulate phases to selectively amplify correct state amplitudes while canceling out error states.

2. ANALYTICAL MECHANICS: THE HAMILTONIAN FORMULATION

Quantum physics discards Newtonian forces ($\vec{F} = m\vec{a}$) in favor of energy optimization. Erwin Schrödinger's equation requires restructuring classical systems into the canonical state space of **Hamiltonian Mechanics**.

The State Map: Systems are tracked in a multi-dimensional Phase Space using generalized coordinates q_i (position) and p_i (momentum). The complete state energy is governed by the scalar function:

$$H(q,p) = T(p) + V(q) = \frac{p^2}{2m} + V(q) \qquad \implies \qquad \text{Hamilton's Equations: } \dot{q} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial q}$$

Quantum Link: Postulate 4 upgrades this classical function directly into a complex differential matrix matrix—the **Hamiltonian Operator** ($\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + V$).

3. NORMAL MODES & COUPLED RESONANCE

When discrete classical oscillators (such as pendulums or LC circuits) are coupled together, their individual, localized movement paths merge into collective, synchronized behaviors called **Normal Modes**.

THE MATRICES OF COUPLED RESONANCE

The equations of motion for a system of coupled harmonic systems can be formulated as a linear eigenvalue matrix problem:

$$\mathbf{M}\ddot{\vec{x}} + \mathbf{K}\vec{x} = 0 \qquad \implies \qquad \det\left(\mathbf{K} - \omega^2\mathbf{M}\right) = 0$$

Quantum Link: The discrete normal mode frequencies (ω_n) represent the system's natural resonances. This behavior forms the classical analog to solving the time-independent Schrödinger Equation for **Energy Eigenstates**. It also models the exact coupling mechanics behind **Rabi Oscillations**.

4. TOTAL INTERNAL REFLECTION & EVANESCENT WAVES

When light transitions from a dense medium (refractive index n_1) to a less dense medium (n_2) at an angle exceeding the critical threshold ($\theta > \theta_c$), it undergoes **Total Internal Reflection (TIR)**.

The Boundary Leakage: While 100% of propagating wave energy reflects backward, satisfying boundary equations forces a non-propagating field to leak across the interface—the **Evanescent Wave**:

$$E(x,t) = E_0\exp(-\kappa x)\exp\left(i(k_y y - \omega t)\right) \qquad \text{where spatial decay maps as } \kappa = \frac{\omega}{c}\sqrt{n_1^2\sin^2\theta - n_2^2}$$

Quantum Link: If a second dense medium is brought close to the boundary, light skips across the air gap. This classical phenomenon, called **Frustrated Total Internal Reflection**, utilizes the exact same exponential boundary equations that govern **Quantum Tunneling** and sub-wavelength field trapping in **Nanophotonics**.