QUANTUM INFORMATION ARCHITECTURE

ESSENTIAL QUANTUM STATES

Properties, Algebraic Structures, and Classification Groups

An axiomatic guide detailing the foundational multi-qubit state configurations that power quantum computational superiority.

1. THE BELL STATES (MAXIMALLY ENTANGLED QUBIT PAIRS)

The four Bell states form a complete orthonormal basis for a two-qubit Hilbert space $\mathcal{H}_4 = \mathcal{H}_2 \otimes \mathcal{H}_2$. They contain the maximum possible amount of non-local correlations allowed by quantum mechanics.

The Bell Basis Vectors:

$$|\Phi^{\pm}\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}} \qquad |\Psi^{\pm}\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$$

Physical Utility: These states serve as the fundamental resource currency for **Quantum Teleportation**, Superdense Coding, and Ekert's entanglement-based cryptography (E91).

2. GREENBERGER-HORNE-ZEILINGER (GHZ) STATES

The GHZ state scales maximally entangled superpositions out to N qubits. It represents a macroscopic "Schrödinger's Cat" state where global quantum coherence spans the entire system uniformly.

Mathematical Formulation: $|\text{GHZ}\rangle_N = \frac{1}{\sqrt{2}}\left(|0\rangle^{\otimes N} + |1\rangle^{\otimes N}\right) = \frac{|00\ldots0\rangle + |11\ldots1\rangle}{\sqrt{2}}$

- **Global Fragility Gotcha:** The coherence is strictly collective. Tracing out or measuring just 1 qubit collapses the entire state into a completely classical mixed product state: $\rho_{\text{reduced}} = \frac{1}{2}|0\ldots0\rangle\langle 0\ldots0| + \frac{1}{2}|1\ldots1\rangle\langle 1\ldots1|$.
- **Metrology Edge:** Provides high phase sensitivity matching the ultimate **Heisenberg Limit** ($\propto 1/N$).

3. THE W STATE (ROBUST SINGLE-EXCITATION SUPERPOSITION)

In stark contrast to the fragile GHZ architecture, the N-qubit **W State** distributes a single quantum of energy (excitation) equally across all components.

ENTANGLEMENT ROBUSTNESS UNDER PARTIAL TRACE

The W state distributes the excitation vector symmetrically across the computational space:

$$|W\rangle_N = \frac{1}{\sqrt{N}}\left(|100\ldots0\rangle + |010\ldots0\rangle + \cdots + |000\ldots1\rangle\right)$$

The Core Advantage: If you lose or measure 1 qubit, the remaining system maintains a highly entangled state with a non-zero residual entanglement property, making it exceptionally robust against localized particle loss.

4. ENTANGLEMENT CLASSIFICATION: GHZ VS. W CLASSES

For $N \geq 3$ qubits, states cannot always be converted into one another using exclusively **Stochastic Local Operations and Classical Communication (SLOCC)**. This splits true multi-qubit entanglement into two fundamentally independent, non-interchangeable resource classes:

1. **GHZ-Class:** Characterized by maximal genuine tri-partite correlations. It represents a true non-local global state but is highly susceptible to erasure errors.
2. **W-Class:** Characterized by highly robust pairwise correlations. It contains no genuine global tri-partite correlations, but retains maximum resistance to channel loss.