THE MICROSCOPIC AND PHENOMENOLOGICAL FOUNDATIONS OF SUPERCONDUCTIVITY

LONDON & BCS THEORIES

Electrodynamics, Electron-Phonon Coupling, and Cooper Pairing

An axiomatic guide detailing macroscopic shielding, microscopic quantum pairing, and the BCS ground state.

1. PHENOMENOLOGICAL FOUNDATIONS: THE LONDON EQUATIONS

Formulated by Heinz and Fritz London in 1935, this framework mathematically explains zero electrical resistance and the **Meissner Effect**—the spontaneous expulsion of internal magnetic fields below a critical temperature T_c.

The Dual London Formulations: Assuming a local number density n_s of superconducting charge carriers with mass m and charge e, the modified Maxwellian kinematics are defined as:

$$\textbf{First London Equation (Zero Resistance):} \quad \frac{\partial \vec{J}_s}{\partial t} = \frac{n_s e^2}{m}\vec{E}$$

$$\textbf{Second London Equation (Meissner Expulsion):} \quad \nabla \times \vec{J}_s = -\frac{n_s e^2}{m}\vec{B}$$

Where: $\vec{J}_s$ is the superconducting current density, $\vec{E}$ is the electric field, and $\vec{B}$ is the magnetic field.

Magnetic Penetration Depth: Substituting the Second London Equation into Ampere's Law ($\nabla \times \vec{B} = \mu_0 \vec{J}_s$) under static conditions yields the Helmholtz profile:

$$\nabla^2 \vec{B} = \frac{1}{\lambda_L^2}\vec{B} \qquad \Longrightarrow \qquad B(x) = B_0 \exp\left(-\frac{x}{\lambda_L}\right) \qquad \text{where } \lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

The field decays exponentially inside the material substrate. This mathematically proves that a superconductor acts as a perfect **diamagnet** ($\vec{B} = 0$ in the bulk), fundamentally distinguishing it from a hypothetical "perfect classical conductor".

2. MICROSCOPIC FOUNDATIONS: THE BCS MODEL (1972 NOBEL PRIZE)

Developed by Bardeen, Cooper, and Schrieffer in 1957, this framework explains the microscopic mechanism of superconductivity. An attractive interaction mediated by lattice vibrations (**phonons**) binds two electrons with opposite momenta and spins into a single composite boson called a **Cooper Pair**.

The Pairing Mechanism: An electron distorts the surrounding positively charged ion lattice, creating a localized region of high positive charge density. A second electron is attracted to this path distortion before the lattice relaxes, overcoming the baseline classical Coulomb repulsion.

3. THE SECOND-QUANTIZED BCS HAMILTONIAN

The effective many-body Hamiltonian modeling this electron-phonon attraction in momentum space restricts pairings strictly to time-reversed electron states ($\vec{k}\uparrow, -\vec{k}\downarrow$):

$$H_{\text{BCS}} = \sum_{\vec{k},\sigma} \xi_{\vec{k}} c^\dagger_{\vec{k}\sigma} c_{\vec{k}\sigma} - \sum_{\vec{k},\vec{k}'} V_{\vec{k}\vec{k}'} c^\dagger_{\vec{k}'\uparrow} c^\dagger_{-\vec{k}'\downarrow} c_{-\vec{k}\downarrow} c_{\vec{k}\uparrow}$$

Where $c^\dagger/c$ are fermionic operators, $\xi_{\vec{k}}$ is the kinetic energy relative to the Fermi energy, and $V_{\vec{k}\vec{k}'}$ is the attractive coupling matrix element.

THE COHERENT BCS GROUND STATE

The global ground state $|\Psi_{\text{BCS}}\rangle$ represents a macroscopically coherent superposition of states with varying total particle numbers, diagonalized using a Bogoliubov transformation:

$$|\Psi_{\text{BCS}}\rangle = \prod_{\vec{k}} \left(u_{\vec{k}} + v_{\vec{k}} c^\dagger_{\vec{k}\uparrow} c^\dagger_{-\vec{k}\downarrow}\right)|0\rangle$$

Where Normalization Demands: $u_{\vec{k}}^2 + v_{\vec{k}}^2 = 1$ and $v_{\vec{k}}^2$ maps pair occupation probability.

The Superconducting Energy Gap: This quantum coherence opens a symmetric energy gap $\Delta_{\vec{k}} = \sum_{\vec{k}'} V_{\vec{k}\vec{k}'} \langle c_{-\vec{k}'\downarrow} c_{\vec{k}'\uparrow}\rangle$ at the Fermi surface. It requires a minimum energy of 2Δ to break a Cooper pair and create single-particle excitations.

4. FUNDAMENTAL LENGTH SCALES: LONDON VS. PIPPARD VS. BCS

The phase dynamics and electrical screening inside a superconductor are completely determined by the interplay between two fundamental internal length scales:

1. **London Penetration Depth (λ_L):** The characteristic distance to which an external magnetic field can penetrate the superconducting boundary.
2. **BCS/Pippard Coherence Length (ξ_0):** The spatial extent or size of a Cooper pair, mathematically bounded by the energy gap: $\xi_0 = \hbar v_F / \pi \Delta$, where v_F is the Fermi velocity.

The Ginzburg-Landau Parameter ($\kappa = \lambda / \xi$): If $\kappa < 1/\sqrt{2}$, the material is a **Type-I** superconductor with an abrupt Meissner breakdown. If $\kappa > 1/\sqrt{2}$, it is a **Type-II** superconductor, allowing magnetic fields to enter through quantized vortices without destroying the global bulk coherence.