★ NOBEL PRIZE WINNING DISCOVERIES IN QUANTUM PHYSICS

MACROSCOPIC QUANTUM EFFECTS

Josephson Relations, Flux Quantization, and Metrology Standards

An axiomatic guide detailing the equations, phase dynamics, and macroscopic wave functions that won the Nobel Prize.

1. THE JOSEPHSON JUNCTION (1973 NOBEL PRIZE)

Predicted by Brian Josephson in 1962, a **Josephson Junction** consists of two superconducting layers separated by a thin insulating barrier ($S - I - S$). This structure allows Cooper pairs to tunnel through without any voltage drop, establishing a single macroscopic wave function spanning the entire system.

The Constitutive Josephson Relations: The macroscopic phase difference $\phi = \phi_2 - \phi_1$ across the insulating barrier determines the current and voltage parameters of the junction:

$$\textbf{DC Josephson Effect:} \quad I(t) = I_c \sin \phi(t)$$

$$\textbf{AC Josephson Effect:} \quad V(t) = \frac{\hbar}{2e}\frac{d\phi(t)}{dt} = \frac{1}{2\pi\Phi_0}\frac{d\phi(t)}{dt}$$

Where: I_c is the maximum critical current, $V(t)$ is the voltage, and $\Phi_0 = h/2e$ is the magnetic flux quantum.

The Core Physical Implications:

- **Constant Voltage Input:** Setting $V(t) = V_{\text{dc}}$ forces the phase to scale linearly in time, creating a high-frequency AC current that oscillates precisely at the Josephson frequency: $\omega_J = \frac{2e}{\hbar}V_{\text{dc}}$.
- **Non-linear Inductance:** Differentiating the DC equation reveals that the junction acts as a non-linear inductor: $L_J(\phi) = \frac{\Phi_0}{2\pi I_c \cos\phi}$. This specific parameter prevents uniform harmonic spacing, isolating the $\{|0\rangle, |1\rangle\}$ states required to construct superconducting qubits.

2. MAGNETIC FLUX QUANTIZATION (1972 NOBEL PRIZE / BCS THEORY)

The single-valued nature of the macroscopic superconducting wave function $\Psi = \sqrt{\rho}e^{i\theta}$ demands that traveling along any closed loop inside a bulk superconductor forces the phase change $\oint \nabla\theta \cdot d\vec{l}$ to be an integer multiple of 2π.

The Magnetic Constraint: This strict geometric phase restriction forces the total magnetic flux Φ threading through a superconducting ring to be perfectly quantized:

$$\Phi = n\Phi_0 = n\frac{h}{2e} \approx n \cdot 2.0678 \times 10^{-15} \text{ Wb} \qquad (n \in \mathbb{Z})$$

*The presence of the factor 2 in the denominator serves as definitive experimental verification that the primary charge carriers inside a superconductor are paired electrons (**Cooper Pairs**).*

3. QUANTUM INTERFERENCE: THE SQUID PLATFORM

A **SQUID** (Superconducting Quantum Interference Device) splices two Josephson junctions in parallel across a closed superconducting loop, combining the AC/DC Josephson relations with macroscopic flux quantization.

MACROSCOPIC WAVE INTERFERENCE

The total critical current traveling through the device exhibits extreme periodic sensitivity to the external magnetic flux Φ_{ext} threading the loop area:

$$I_{\max} = 2I_c \left|\cos\left(\pi\frac{\Phi_{\text{ext}}}{\Phi_0}\right)\right|$$

The Measurement Boundary: Because Φ_0 is tiny, tracking these periodic shifts allows SQUIDs to detect magnetic fields down to the sub-femtotesla scale ($< 10^{-15}$ T), mapping neural fields or tracking localized quantum spin flips.

4. PRECISION METROLOGY AND QUANTUM VOLTAGE STANDARDS

The AC Josephson effect relies exclusively on fundamental physical constants (e and h). This independence makes it the global tracking standard for calibrating electrical potential.

1. **The Josephson Constant:** Defined as $K_J = 2e/h \approx 483,597.89$ GHz/V.

2. **Shapiro Steps:** Irradiating a junction with a precise microwave frequency f locks the internal phase oscillations, generating discrete, error-immune voltage steps: $V_n = n\frac{f}{K_J}$. This removes all material dependencies from international standard calibrations.