QUANTUM SHANNON THEORY & CAPACITY CONSTRAINTS

# QUANTUM INFORMATION BOUNDS

## Accessible Information, Entanglement Monotones, and Transmission Limits

An axiomatic guide detailing the fundamental inequalities, mathematical bounds, and limits of quantum information transmission.

## 1. THE HOLEVO BOUND (FUNDAMENTAL ACCESSIBLE INFORMATION LIMIT)

Formulated by Alexander Holevo in 1973, this theorem establishes an upper bound on the amount of classical information that can be extracted from an ensemble of quantum states. It proves that despite a qubit existing in a continuous, infinite-dimensional superposition space, encoding classical data into $N$ qubits can yield at most $N$ classical bits of readable information.

**The Holevo Quantity ($\chi$) Formula:** Let Alice prepare a classical message $x$ with probability $p_x$, mapping it to a quantum density matrix $\rho_x$. The global averaged mixed state is $\rho = \sum_x p_x \rho_x$. The maximum classical accessible information $I(X:Y)$ extracted by Bob across any measurement configuration is upper-bounded by:

$$I(X:Y) \leq \chi = S(\rho) - \sum_x p_x S(\rho_x)$$

**Where:** $S(\rho) = -\mathrm{Tr}(\rho \log_2 \rho)$ represents the Von Neumann entropy function.
✦ **The Core Constraint:** If all state options in the ensemble are mutually orthogonal ($\rho_i \rho_j = 0$), $\chi$ matches the classical Shannon entropy exactly. If the states are non-orthogonal, $\chi$ drops strictly lower, preventing perfect message discrimination.

## 2. THE TSIRELSON BOUND (NON-LOCAL CORRELATION LIMITS)

Discovered by Boris Tsirelson in 1980, this limit defines the absolute boundary of quantum non-locality. While local hidden variable theories are restricted by classical local realism inequalities, quantum mechanics permits stronger correlations, which remain strictly bounded to prevent the violation of relativistic causality.

**The Mathematical CHSH Evaluation:** For an experimental setup evaluating two spatially separated particle nodes under binary measurement operators $A_0, A_1$ and $B_0, B_1$:

$$\text{Correlation Parameter: } S = \langle A_0 B_0 \rangle + \langle A_0 B_1 \rangle + \langle A_1 B_0 \rangle - \langle A_1 B_1 \rangle$$

**Local Realism Boundary:** $|S| \leq 2$ $\longleftrightarrow$ **Tsirelson Quantum Bound:** $\mathbf{|S| \leq 2\sqrt{2} \approx 2.828}$

*Hypothetical post-quantum theories (such as PR-Boxes) allow correlations up to the absolute algebraic maximum of $S = 4$, but their absence in nature proves that the universe restricts non-local interactions exactly at Tsirelson's boundary.*

## 3. BREMERMANN'S LIMIT  THE MARGOLUS-LEVITIN THEOREM

To scale quantum processors or transmission pipelines, information processing velocities are bounded by the fundamental energy constraints of quantum physics.

### MAXIMUM COMPUTATIONAL QUANTUM VELOCITY

The Margolus-Levitin Theorem dictates the absolute minimum time $\tau_{\min}$ required for a quantum state to evolve from its initial state into an orthogonal state configuration:

$$\tau_{\min} \geq \frac{h}{4(E - E_0)} = \frac{\pi \hbar}{2 \Delta E}$$

**The Physics Boundary:** Combined with Bremermann's Limit, this defines the ultimate data processing capacity for any material system: a maximum processing speed limit bounded at $\mathbf{C \approx 2 \times 10^{47}}$ bits/sec/gram.

## 4. ENTANGLEMENT MONOTONE BOUNDS (MONOGAMY OF ENTANGLEMENT)

Quantifying multi-partite systems requires evaluating constraints on how non-local resources scale. Coffman, Kundu, and Wootters (CKW) established that entanglement behaves like a monogamous resource: it cannot be infinitely shared among independent parties.

**The CKW Monogamy Inequality:** For a three-party system $ABC$, let $\tau(A|B)$ represent the squared concurrence measuring the explicit entanglement isolated between node $A$ and node $B$:

$$\tau(A|B) + \tau(A|C) \leq \tau(A|BC)$$