QUANTUM OPTICS, FIELD QUADRATURES, & PHOTONIC PHASE SPACE

QUANTUM STATES OF LIGHT

Fock Baselines, Coherent Fields, Squeezed Vacuum, and Phase Space Quadratures

An axiomatic mathematical flyer defining the algebraic structures and uncertainty profiles of quantized optical fields.

1. THE FOUNDATION: FIELD OPERATORS AND PHASE SPACE QUADRATURES

When an electromagnetic field mode is quantized, its classical electric field vector is upgraded to a linear combination of non-Hermitian bosonic ladder operators $(a, a^\dagger)$. To map this continuous phase space, the field is broken into dimensionless, Hermitian **Field Quadratures**:

Position-like (Amplitude) Quadrature: $X_1 = \dfrac{a + a^\dagger}{2}$ **Momentum-like (Phase) Quadrature:** $X_2 = \dfrac{a - a^\dagger}{2i}$

The canonical bosonic commutation relation $[a, a^\dagger] = I$ forces the quadratures to behave as non-commuting variables. This creates a fundamental uncertainty lower bound for optical measurements:

$$[X_1, X_2] = \frac{i}{2} \qquad \Longrightarrow \qquad \Delta X_1 \Delta X_2 \geq \frac{1}{4}$$

2. FOCK STATES (NUMBER STATES)

Fock states $|n\rangle$ represent states with a precisely defined, discrete number of photons. They form a complete, orthonormal basis for a single-mode optical Hilbert space.

Algebraic Properties and Shot-Noise Profiles: A Fock state is an eigenstate of the number operator: $N|n\rangle = a^\dagger a|n\rangle = n|n\rangle$.
✦ **Perfect Intensity Certainty:** The variance of the photon count drops to zero: $\Delta N^2 = \langle N^2 \rangle - \langle N \rangle^2 = 0$.
✦ **Total Phase Uncertainty:** Because intensity and phase are conjugate variables, a state with absolute photon certainty carries completely randomized phase characteristics. Its phase-space profile maps as a uniform symmetric ring.

3. COHERENT STATES (THE SEMI-CLASSICAL LIMIT)

Generated natively by stable single-mode lasers, a **Coherent State** $|\alpha\rangle$ is the closest quantum mechanical approximation to a classical sinusoidal wave. It is mathematically formulated as the eigenstate of the non-Hermitian annihilation operator: $a|\alpha\rangle = \alpha|\alpha\rangle$, where $\alpha = |\alpha|e^{i\theta} \in \mathbb{C}$.

The Displacement Operator: Generated from the vacuum state $|0\rangle$ via a unitary mapping: $|\alpha\rangle = D(\alpha)|0\rangle = \exp(\alpha a^\dagger - \alpha^* a)|0\rangle$.

✦ **Poissonian Photon Statistics:** The probability of measuring exactly n photons follows a Poisson distribution: $P(n) = \frac{|\alpha|^{2n}}{n!} e^{-|\alpha|^2}$. The photon count variance matches the mean intensity: $\Delta N^2 = \langle N \rangle = |\alpha|^2$.
✦ **Symmetric Minimal Uncertainty Bounds:** Coherent states maintain identical minimum uncertainty across both phase-space quadratures indefinitely: $\Delta X_1 = \Delta X_2 = 1/2 \implies \Delta X_1 \Delta X_2 = 1/4$.

4. SQUEEZED STATES OF LIGHT (BYPASSING VACUUM NOISE BOUNDARIES)

A **Squeezed State** bypasses the symmetric noise constraints of coherent fields. It utilizes non-linear optical processes (such as parametric down-conversion) to compress the quantum noise of one chosen quadrature below the standard vacuum limit ($\Delta X_i < 1/2$), at the necessary cost of exponentially amplifying the noise in the conjugate quadrature.

THE UNITARY SQUEEZING TRANSFORMATION

Squeezed states are generated by applying the two-photon Unitary Squeezing Operator $S(\xi)$ to the vacuum state:

$$|\xi\rangle = S(\xi)|0\rangle = \exp\left(\frac{1}{2}\xi^* a^2 - \frac{1}{2}\xi (a^\dagger)^2\right)|0\rangle \qquad \text{where the scaling parameter is: } \xi = re^{i\theta}$$

The Quadrature Noise Imbalance: For $\theta = 0$, the noise profiles scale as: $\mathbf{\Delta X_1 = \frac{1}{2}e^{-r}}$ and $\mathbf{\Delta X_2 = \frac{1}{2}e^{r}}$.
The Metrology Edge: The compressed quadrature satisfies $\Delta X_1 \ll 1/2$ while preserving the absolute minimum uncertainty product: $\Delta X_1 \Delta X_2 = 1/4$. This precise noise reduction is deployed inside laser interferometers (such as **LIGO**) to measure gravitational space-time distortions below the standard classical quantum limit.

5. COMPARATIVE SUMMARY OF PRIMARY STATE MODALITIES

State Class	Photon Variance (ΔN^2)	Quadrature Profile	Primary Physical Resource Application
Fock State $\lvert n\rangle$	0 (Sub-Poissonian)	Circular Ring	Quantum Cryptography, Single-Photon Sources
Coherent $\lvert \alpha\rangle$	$\langle N\rangle$ (Poissonian)	Symmetric Disk	Telecommunications, Classical Free-Space Optics
Squeezed $\lvert \xi\rangle$	Super-Poissonian	Stretched Ellipse	Quantum Metrology, Phase-Sensitive Detection