ELECTRODYNAMICS, MACROSCOPIC COHERENCE, & QUANTUM PHASE TRANSITIONS

# SUPERCONDUCTING FEATURES

## The Meissner Effect, Magnetic Flux Quantization, and Phase Coherence Boundaries

An axiomatic mathematical guide detailing the macroscopic thermodynamic traits and field constraints of superconducting phases.

## 1. THE FOUNDATIONAL PARADOX: PERFECT CONDUCTOR VS. SUPERCONDUCTOR

A common conceptual trap is defining a superconductor merely as a classical conductor with zero electrical resistance ($\sigma \to \infty$). Ohm's Law ($\vec{J} = \sigma \vec{E}$) dictates that an ideal classical conductor frozen under zero-resistance conditions traps its initial internal magnetic flux indefinitely ($d\vec{B}/dt = 0$). True superconductivity breaks this classical memory constraint via the **Meissner-Ochsenfeld Effect**: a superconductor actively and spontaneously expels all interior magnetic fields when cooled below its critical temperature $T_c$, behaving as a perfect thermodynamic diamagnet regardless of its initial state.

## 2. MATHEMATICAL FORMULATION OF THE MEISSNER EFFECT

The active shielding current loops responsible for field expulsion are governed by the phenomenological **London Equations**. Combining the acceleration of superconducting charge carriers with Maxwell's vector field kinematics yields:

**The London Electrodynamic System:** Let $n_s$ represent the local number density of paired superconducting electrons (mass $m$, charge $e^* = 2e$). The current density $\vec{J}_s$ satisfies:

$$\textbf{First London Equation (Zero Resistance):} \quad \frac{\partial \vec{J}_s}{\partial t} = \frac{n_s (e^*)^2}{m} \vec{E}$$

$$\textbf{Second London Equation (Flux Expulsion):} \quad \nabla \times \vec{J}_s = -\frac{n_s (e^*)^2}{m} \vec{B}$$

Substituting the second equation into Ampere's Law ($\nabla \times \vec{B} = \mu_0 \vec{J}_s$) yields the field screening boundary equation: $\nabla^2 \vec{B} = \frac{1}{\lambda_L^2} \vec{B}$

Solving this vector Helmholtz equation for a flat boundary proves that magnetic fields cannot survive within the bulk material, dropping exponentially past the interface:

$$B(x) = B_0 \exp\left(-\frac{x}{\lambda_L}\right) \qquad \text{where the } \textbf{London Penetration Depth} \text{ is } \lambda_L = \sqrt{\frac{m}{\mu_0 n_s (2e)^2}}$$

## 3. MACROSCOPIC QUANTUM COHERENCE: FLUX QUANTIZATION

Superconductivity links microscopic quantum mechanics to macroscopic variables. The entire collection of Cooper pairs behaves as a single coherent quantum state vector defined by a macroscopic wave function $\Psi(\vec{r}) = \sqrt{n_s(\vec{r})} e^{i\theta(\vec{r})}$. Because this wave function must be single-valued, tracking the phase gradient $\nabla\theta$ along any closed loop inside a bulk superconductor restricts the total integrated path value:

### THE MAGNETIC FLUX QUANTUM BOUNDARY

Evaluating the gauge-invariant quantum current density $\vec{J}_s = \frac{2e\hbar}{m}\left(\nabla\theta - \frac{2e}{\hbar}\vec{A}\right)$ along a closed path deep in the bulk (where $\vec{J}_s = 0$) forces the phase change to match an integer multiple of $2\pi$:

$$\oint \nabla\theta \cdot d\vec{l} = 2\pi n \quad \Longrightarrow \quad \Phi = \oint \vec{A} \cdot d\vec{l} = n\left(\frac{h}{2e}\right)$$

**The Metric Scale:** The trapped magnetic field lines are restricted to discrete, fundamental units called the **Magnetic Flux Quantum**: $\boldsymbol{\Phi_0 = \frac{h}{2e} \approx 2.0678 \times 10^{-15}}$ Wb. The factor 2 provides direct proof that the carriers are Cooper pairs.

## 4. PHASE BOUNDARIES: TYPE-I VS. TYPE-II TOPOLOGIES

Superconducting states are bounded by their surface energy conditions. The relationship between the London penetration depth ($\lambda_L$) and the **BCS Coherence Length** ($\xi_0$, which maps the physical size of a Cooper pair) determines how a material breaks down under external magnetic fields:

- **Type-I Superconductors ($\kappa = \lambda_L/\xi_0 < 1/\sqrt{2}$):** Exhibit an abrupt phase collapse. The Meissner effect is perfectly preserved until the field passes a singular critical threshold $H_c$, where the entire bulk instantly returns to a normal, resistive state.
- **Type-II Superconductors ($\kappa = \lambda_L/\xi_0 > 1/\sqrt{2}$):** Support a multi-stage breakdown. Between a lower critical field $H_{c1}$ and an upper critical field $H_{c2}$, the material enters the **Mixed (Vortex) State**.

**The Abrikosov Vortex Lattice:** In the mixed state, magnetic fields pierce the superconductor by forming thin filaments called **fluxons**. Each filament holds exactly one unit of flux $\Phi_0$ and is wrapped in a localized supercurrent vortex. The surrounding bulk space remains perfectly superconducting, allowing Type-II systems to survive massive magnetic fields ($> 100$ Tesla) for industrial power applications.