FOUNDATIONS AND FOUNDATIONS OF QUANTUM NON-LOCALITY

THE EPR PARADOX

Local Realism vs. Quantum Completeness

An axiomatic breakdown of the 1935 Einstein-Podolsky-Rosen thought experiment and its resolution.

1. THE CORE PREMISE: CRITERIA FOR REALITY

In 1935, Einstein, Podolsky, and Rosen (EPR) challenged the completeness of quantum mechanics. They asserted that a physical theory must satisfy two fundamental philosophies to be considered complete:

- **Locality:** Physical actions happening at point A cannot instantly affect physical reality at a spatially separated point B.
- **Realism:** If the value of a physical quantity can be predicted with certainty without disturbing the system, there exists an **element of physical reality** corresponding to it.

2. THE BOHM VARIANT: ENTANGLED SINGLET STATE

David Bohm simplified the EPR thought experiment using a pair of entangled spin-1/2 particles generated from a single decaying source. The composite system is described by the maximally entangled singlet state:

The Singlet State Vector:

$$|\psi^-\rangle = \frac{|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle}{\sqrt{2}} = \frac{|+\rangle_x|-\rangle_x - |-\rangle_x|+\rangle_x}{\sqrt{2}}$$

The global quantum coherence ensures perfect anti-correlation along **any** chosen measurement axis (z, x, or an arbitrary direction $\hat{n}$).

3. THE PARADOXICAL LOOP

If Alice measures her particle along the z-axis and observes spin-up ($|\uparrow\rangle$), the global state instantly collapses. She knows with 100% certainty, without touching Bob's particle, that Bob will measure spin-down ($|\downarrow\rangle$).

1. By the definition of **Realism**, Bob's z-spin must be an objective intrinsic reality before his measurement.
2. By the definition of **Locality**, Alice's remote action could not have created this reality.
3. **The Catch:** If Alice had instead measured along the x-axis, she would have instantly determined Bob's x-spin reality. Since X and Z matrices do not commute ($[X, Z] \neq 0$), quantum mechanics forbids both values from existing simultaneously.

EPR Conclusion: Since both properties can be made real via choice of measurement, but quantum states cannot describe both at once, quantum mechanics must be an **incomplete theory** requiring hidden variables.

4. THE QUANTITATIVE RESOLUTION: BELL'S THEOREM

In 1964, John Stewart Bell proved that the assumptions of local realism are mathematically testable. No local hidden variable theory can reproduce all predictions of quantum mechanics.

VIOLATION OF LOCAL REALISM

Evaluating statistical correlations under arbitrary angles a and b yields the CHSH inequality. Local realism demands a strict boundary on the correlation parameter S:

$$\text{Local Realism Constraint: } |S| \leq 2 \quad \longleftrightarrow \quad \text{Quantum Mechanical Reality: } |S| = 2\sqrt{2} \approx 2.828$$

Decades of loophole-free experimental tests definitively confirm the quantum bound, rejecting local realism.

5. MODERN QUANTUM INFORMATION CONTEXT

The EPR paradox is no longer viewed as a flaw in quantum theory, but as the foundational validation of quantum non-locality. The global coherence inherent in EPR pairs forms the operational basis for modern applications such as **Quantum Teleportation**, Superdense Coding, and Entanglement-Based QKD (E91).