FUNDAMENTAL BOUNDARIES OF NATURE

THE LAWS OF QUANTUM PHYSICS

Universal Constraints and Non-Classical Limit Theorems

An axiomatic guide to the fundamental laws, conservation constraints, and impossibility theorems of quantum mechanics.

1. THE LAW OF QUANTUM FLUCTUATIONS (HEISENBERG UNCERTAINTY)

Physical observables are governed by non-commutative algebra. This establishes a fundamental lower bound on the simultaneous precision of conjugate physical properties, proving that uncertainty is a native property of nature, not an experimental limitation.

The Robertson-Schrödinger Uncertainty Relation: For any two arbitrary Hermitian operators (observables) A and B:

$$\sigma_A^2 \sigma_B^2 \geq \left| \frac{1}{2i} \langle [A, B] \rangle \right|^2 \quad \xrightarrow{\text{Position \& Momentum}} \quad \sigma_x \sigma_p \geq \frac{\hbar}{2}$$

Where $[A, B] = AB - BA$ is the commutator and σ represents the standard deviation.

2. THE LAW OF INFORMATION CONSERVATION (NO-CLONING THEOREM)

Derived by Wootters, Zurek, and Dieks in 1982, this law states that it is physically impossible to create an identical, independent copy of an arbitrary, unknown quantum state.

The Proof: Assume a unitary operator U exists such that it clones state $|\psi\rangle$ and $|\phi\rangle$ onto a target blank state $|e\rangle$:

$$U |\psi\rangle |e\rangle = |\psi\rangle |\psi\rangle \quad \text{and} \quad U |\phi\rangle |e\rangle = |\phi\rangle |\phi\rangle$$

Taking the inner product of both expressions yields:

$$\langle \psi|\phi\rangle \langle e|e\rangle = \langle \psi|\phi\rangle \langle \psi|\phi\rangle \implies \langle \psi|\phi\rangle = (\langle \psi|\phi\rangle)^2$$

The Impossibility Constraint: This equation only holds true if $\langle \psi|\phi\rangle = 0$ or 1. Therefore, cloning is exclusively permitted for predefined, mutually orthogonal states, preventing the classical copying of quantum information.

3. THE LAW OF CAUSALITY (NO-COMMUNICATION THEOREM)

This law guarantees that despite the presence of instantaneous, non-local collapse in entangled states (like the EPR singlet or GHZ state), quantum mechanics cannot be used to transmit classical information faster than the speed of light (c).

MATHEMATICAL CAUSALITY IN MIXED STATES

Let Alice and Bob share a joint density operator ρ_{AB}. If Alice performs a local measurement using a set of projection operators $\{P_i^A\}$, Bob's isolated local state is found by taking the partial trace:

$$\rho_B' = \mathrm{Tr}_A \left(\sum_i (P_i^A \otimes I) \rho_{AB} (P_i^A \otimes I)^\dagger \right) = \mathrm{Tr}_A(\rho_{AB}) = \rho_B$$

Because Bob's reduced density matrix remains completely unchanged by Alice's choice of measurement, her remote action is locally undetectable to Bob without a sub-lightspeed classical signal.

4. THE LAW OF IDENTICAL PARTICLES (SPIN-STATISTICS THEOREM)

In quantum mechanics, identical particles are fundamentally indistinguishable. The global multi-body wave function must update symmetrically under the permutation operator P_{12} that swaps particle 1 and particle 2:

- **Bosons (Integer Spin):** Symmetric state evolution. $P_{12} |\psi_1 \psi_2\rangle = + |\psi_2 \psi_1\rangle$.
- **Fermions (Half-Integer Spin):** Anti-symmetric state evolution. $P_{12} |\psi_1 \psi_2\rangle = - |\psi_2 \psi_1\rangle$.

The Pauli Exclusion Principle: If two identical fermions occupy the exact same quantum state $|\phi\rangle$, anti-symmetrization forces the state vector to zero: $|\phi\rangle |\phi\rangle = - |\phi\rangle |\phi\rangle \implies \mathbf{2} |\phi\rangle |\phi\rangle = \mathbf{0}$. No two fermions can ever occupy identical quantum states simultaneously.