COMPLEMENTARITY, ENTANGLEMENT, AND RETROCAUSAL PARADOXES

THE QUANTUM ERASER

Restoring Wave Interference via Information Destruction

An axiomatic guide detailing the algebraic states, which-way marking, and joint coincidence correlations of quantum erasers.

1. THE COMPLEMENTARITY PRINCIPLE & WHICH-WAY MARKING

Niels Bohr's **Complementarity Principle** dictates that wave-like and particle-like behaviors are mutually exclusive. In a double-slit setup, obtaining "which-way" (welcher-weg) information about a photon's path completely obliterates the spatial interference pattern.

The Entangled Path State Layout: Let a photon pass through a slit system where its path states $|\psi_A\rangle$ (Slit A) and $|\psi_B\rangle$ (Slit B) are perfectly entangled with the orthogonal states of a path detector (or photon polarization tags), denoted as $|A\rangle_D$ and $|B\rangle_D$:

$$|\Psi\rangle_{\text{total}} = \frac{1}{\sqrt{2}} \left(|\psi_A\rangle |A\rangle_D + |\psi_B\rangle |B\rangle_D \right) \qquad \text{where } \langle A|B\rangle_D = 0$$

The Destruction of Interference: Calculating the localized intensity distribution on the screen requires taking the partial trace to eliminate the detector space, resulting in a mixed density matrix:

$$\rho_{\text{screen}} = \text{Tr}_D(|\Psi\rangle \langle\Psi|) = \frac{1}{2} |\psi_A\rangle \langle\psi_A| + \frac{1}{2} |\psi_B\rangle \langle\psi_B|$$

Because the cross-interference terms ($|\psi_A\rangle \langle\psi_B|$) drop to zero due to detector orthogonality, the screen displays a purely classical, featureless Gaussian distribution.

2. THE ERASURE MECHANISM: PROJECTION ONTO A DIAGONAL BASIS

The **Quantum Eraser** proves that the loss of interference is not caused by mechanical momentum kicks from the detector (shattering Heisenberg's original classical intuition). If you permanently erase the which-way information stored inside the detector *before* reading it, the wave interference pattern instantly returns.

The Diagonal Projection: To erase the path memory, the detector qubits are measured across a non-orthogonal diagonal basis: $|\pm\rangle_D = \frac{1}{\sqrt{2}}(|A\rangle_D \pm |B\rangle_D)$. Rewriting the global state vector in terms of this erasing basis yields:

$$|\Psi\rangle_{\text{total}} = \frac{1}{2} \left[(|\psi_A\rangle + |\psi_B\rangle) |+\rangle_D + (|\psi_A\rangle - |\psi_B\rangle) |-\rangle_D \right]$$

- **Outcome (+) Measurement:** Projects the photon into the symmetric state: $|\psi_+\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle + |\psi_B\rangle)$, reconstructing a clean, high-visibility fringe pattern.
- **Outcome (−) Measurement:** Projects the photon into the anti-symmetric state: $|\psi_-\rangle = \frac{1}{\sqrt{2}}(|\psi_A\rangle - |\psi_B\rangle)$, generating an anti-fringe pattern (shifted by a phase of π).

3. THE DELAYED-CHOICE PARADIGM (SCULLY-DRÜHL EXPERIMENT)

In a **Delayed-Choice Quantum Eraser**, the erasing measurement is executed on an entangled idler photon at a spatial distance so vast that it occurs *after* the signal photon has already hit the recording screen.

THE COINCIDENCE CORRELATION RESOLUTION

Because the global screen density matrix remains a uniform mixture ($\rho_{\text{screen}} = \frac{1}{2} |\psi_+\rangle \langle\psi_+| + \frac{1}{2} |\psi_-\rangle \langle\psi_-|$), the raw screen display never shows interference. This protects the no-communication theorem and preserves causality.

$$\text{Total Pattern: } \mathbf{I}_{\text{total}}(\mathbf{x}) = \mathbf{I}_+(\mathbf{x}) + \mathbf{I}_-(\mathbf{x})$$

The Core Fact: The fringes (I_+) and anti-fringes (I_-) complement each other perfectly, washing out the raw image. The quantum erasure can only be extracted retroactively by sorting the screen positions in correlation with the idler detection events using **coincidence counting circuits**.

4. ARCHITECTURAL PIPELINE OF THE KIM ET AL. EXPERIMENT

The classic 1999 deployment utilizes Spontaneous Parametric Down-Conversion (SPDC) and a prism network to route entangled streams:

1. **SPDC Generation:** An ultraviolet laser hits double slits, creating entangled pairs: $|\Psi\rangle \propto |\text{slit}_1\rangle_s |\text{slit}_1\rangle_i + |\text{slit}_2\rangle_s |\text{slit}_2\rangle_i$.
2. **Signal Path:** Signal photons (s) head directly to detector D_0 to record impact coordinates.
3. **Idler Path:** Idler photons (i) route through 50/50 beam splitters. Reaching detectors D_1 or D_2 completely erases which-way histories, while reaching D_3 or D_4 preserves the original path trajectory.