FOUNDATIONS OF QUANTUM MECHANICS

THE QUANTUM POSTULATES

The Mathematical Framework of Quantum Realities

An axiomatic summary mapping physical systems to complex Hilbert spaces.

POSTULATE 1: THE STATE SPACE

Any isolated physical system is completely described by its state vector $|\psi\rangle$, which is a unit vector inside a complex vector space with an inner product (**Hilbert Space**, $\mathcal{H}$).

Normalization Condition:
The superposition principle states that if $|\psi_1\rangle$ and $|\psi_2\rangle$ are valid system states, any linear combination $|\psi\rangle =$ $\alpha |\psi_1\rangle + \beta |\psi_2\rangle$ is also a valid state ($\alpha, \beta \in \mathbb{C}$).

POSTULATE 2: PHYSICAL OBSERVABLES

Every physically measurable property (observable) corresponds to a **Hermitian operator** A acting on the Hilbert space $\mathcal{H}$.

- **Real Eigenvalues:** Spectral theorem guarantees eigenvalues a_n are real ($A|n\rangle = a_n|n\rangle$), matching real physical measurements.
- **Complete Basis:** Eigenvectors form an orthonormal basis, allowing any state to be written as: $|\psi\rangle = \sum_n c_n |n\rangle$.

POSTULATE 3: QUANTUM MEASUREMENT (BORN'S RULE)

Quantum measurements are probabilistic. Measuring an observable A on a state $|\psi\rangle$ projects the state onto the clean eigenspace of the operator.

Probability of Outcome a_n**:** $P(a_n) = |\langle n|\psi\rangle|^2 = \langle\psi| P_n |\psi\rangle$ *where* $P_n = |n\rangle\langle n|$ *is the projection operator.*

STATE COLLAPSE MECHANISM

Immediately following a measurement yielding outcome a_n, the system instantly collapses into the normalized eigenvector:

$$|\psi'\rangle = \frac{P_n |\psi\rangle}{\sqrt{\langle\psi| P_n |\psi\rangle}} = |n\rangle$$

POSTULATE 4: TIME EVOLUTION

The continuous time evolution of an isolated quantum state $|\psi(t)\rangle$ is governed by the **Time-Dependent Schrödinger Equation**.

$$i\hbar \frac{d}{dt} |\psi(t)\rangle = H(t) |\psi(t)\rangle$$

Where $H(t)$ is the Hermitian Hamiltonian operator representing the total energy of the physical system.

For a time-independent Hamiltonian, evolution is strictly deterministic and preserved by a **Unitary Operator** $U(t, t_0)$:

$$|\psi(t)\rangle = U(t, t_0) |\psi(t_0)\rangle = \exp\left(-\tfrac{iH(t-t_0)}{\hbar}\right) |\psi(t_0)\rangle \qquad \textbf{Property: } U^\dagger U = I$$

POSTULATE 5: COMPOSITE SYSTEMS

The state space of a composite physical system containing multiple independent subsystems is the **tensor product** ($\otimes$) of the individual component Hilbert spaces.

$$\mathcal{H}_{\text{total}} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_N$$

If system 1 is in state $|\phi\rangle$ and system 2 is in state $|\chi\rangle$, the joint uncorrelated state is $|\phi\rangle \otimes |\chi\rangle$. Global states that cannot be factored into product states of individual components are natively **entangled**.